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COMMENT

Comment on 'Large-N theory of strongly commensurate dirty bosons: absence of a transition in two dimensions'

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Abstract

We show that the strongly commensurate dirty boson system must have a phase transition in two dimensions, and argue that the low-lying states have a diverging localization length.

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Recently it was claimed [1] that the large-N dirty boson system does not have a phase transition in 2+1 dimensions, and it was conjectured that an $O(3)$ system might not have a phase transition. However, both claims violate correlation inequalities. The authors consider the action

$$S(\Psi) = \int \mathrm{d}^2x\,\mathrm{d}\tau \left\{ (\partial_\tau \psi(x,\tau))^2 + (\partial_x \psi(x,\tau))^2 + (V(x)-\mu)\psi^2(x,\tau) + \frac{\lambda}{N}\psi^4(x,\tau) \right\} \quad (1)$$

with ψ an N-component field, $\psi^2 = \sum \psi_\alpha^2$, and $V(x)$ some random potential. While $V(x)$ was assumed Gaussian, we first consider the more physical case in which $V(x)$ is bounded: $-\delta < V(x) < \delta$, and only later consider the Gaussian case. In order to make use of previous rigorous results, we adopt a lattice regularization including a discretization in the τ direction, obtaining a model of $O(N)$ spins with statistical weight

$$\exp\left(J_{i,j}\psi_\alpha(i)\psi_\alpha(j) - \frac{\lambda}{N}\psi^4(i,\tau) \right) \quad (2)$$

where $J_{i,i} = (\mu - V(i))$[1]. Here, i lies in a three-dimensional lattice, but $V(i)$ is a function of only two of the coordinates of i. This lattice regularization is equivalent to that used in [1].

[1] We note that the term μ in equation (2) differs by a constant from that used in equation (1). This constant difference arises because we have regularized the gradient term in equation (1) by a lattice coupling, $J_{i,j}$; we must then shift the constant μ by the bandwidth of the lattice coupling.

Then,

$$\frac{\partial\langle\psi_\alpha(i)\psi_\alpha(j)\rangle}{\partial J_{kl}} = \langle\psi_\alpha(i)\psi_\alpha(j)\psi_\beta(k)\psi_\beta(l)\rangle - \langle\psi_\alpha(i)\psi_\alpha(j)\rangle\langle\psi_\beta(k)\psi_\beta(l)\rangle \geqslant 0. \tag{3}$$

This inequality is a correlation inequality or Griffiths inequality. Such inequalities were first proven for $N = 1$ [2], later for $N = 2$ [3] and $N = \infty$ [4], and only very recently for all N [5]. These inequalities express the natural result that increasing bond strengths in an unfrustrated system increases the correlations.

If we fix $V(i) = \delta$ for all i, we have a pure system, for which there must be a $\mu_{\mathrm{c,pure}}$ with a ferromagnetic phase for $\mu > \mu_{\mathrm{c,pure}}$. As the $V(i)$ in the random system are all bounded by δ, the random system is obtained from the pure system by *increasing* some of the J_{ij} and thus, by inequality (3), must be in the ferromagnetic phase for $\mu > \mu_{\mathrm{c,pure}}$ and so must have a phase transition at $\mu = \mu_{\mathrm{c,random}}$ with $\mu_{\mathrm{c,random}} \leqslant \mu_{\mathrm{c,pure}}$ (and $\mu_{\mathrm{c,random}} \geqslant \mu_{\mathrm{c,pure}} - 2\delta$).

We can proceed further in the the large-N limit, which is solved by a Hubbard–Stratanovich transformation on the quartic term, and then a saddle point on the Hubbard–Stratanovich field, $\chi(x)$. Returning to the continuum theory, the correlations of ψ are obtained from a noninteracting theory with action

$$S(\Psi) = \int \mathrm{d}^2x\,\mathrm{d}\tau\,\left\{(\partial_\tau\psi(x,\tau))^2 + (\partial_x\psi(x,\tau))^2 + (V(x) - \mu + \chi(x))\psi^2(x,\tau)\right\}. \tag{4}$$

Diagonalizing this action, we denote the lowest eigenvalue by ϵ_0, with normalized eigenvector $\psi_0(x)$. The correlation function $\langle\psi_\alpha(x_1,\tau_1)\psi_\alpha(x_2,\tau_2)\rangle$ is equal to the Green's function of this action, $G(x_1,\tau_1,x_2,\tau_2)$. Since $\epsilon_0^{-1} = \lim_{M\to\infty}(\mathrm{Trace}(G^M))^{1/M}$, the correlation inequalities show that ϵ_0 decreases as J_{ij} is increased and in the infinite volume limit ϵ_0 vanishes for the dirty boson system with $\mu > \mu_{\mathrm{c,pure}}$. This differs from the result of [1], where it was claimed on the basis of finite size scaling of numerical results that the lowest eigenvalue does not vanish for the disordered system. Since the present result is based on correlation inequalities, this probably indicates that finite size scaling does not apply, at least for the system sizes considered in [1].

Further, in the ferromagnetic phase, under the reasonable assumption that there is only one vanishing eigenvalue, $\langle\psi(x)\rangle = \psi_0/\epsilon_0$. The vanishing eigenvalue signals the appearance of spontaneous symmetry breaking within the large-N limit; the eigenvector associated with this eigenvalue is proportional to the expectation value of ψ. Then the correlation inequalities show that $\psi_0(x)$, considered as a function of x for fixed μ, is bounded above and below in the ferromagnetic phase and hence delocalized. We then rewrite the action (4) as [6]

$$S(f(x)) = \int \mathrm{d}^2x\,\mathrm{d}\tau\;\psi_0(x)^2\left\{(\partial_\tau f(x,\tau))^2 + (\partial_x f(x,\tau))^2\right\} \tag{5}$$

where $\psi(x) = f(x)\psi_0(x)$. This action (5) gives a system with random stiffness, for which the localization length diverges at low energies [7], in contrast to the case of on-site disorder.

Finally, we return to Gaussian disorder. We do not know of rigorous results for this case, but if $\mu >> \mu_{\mathrm{c,pure}}$, the probability of any site having $\mu + V(x) \lesssim \mu_{\mathrm{c,pure}}$ is exponentially small, so that the existence of a phase transition seems guaranteed on physical grounds.

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References

[1] Case M J and Herbut I F 2001 *J. Phys. A: Math. Gen.* **34** 7739

[2] Griffiths R B 1967 *J. Math. Phys.* **8** 478
Griffiths R B 1967 *J. Math. Phys.* **8** 484
[3] Ginibre J 1970 *Commun. Math. Phys.* **16** 310
[4] Sylvester G S 1980 *Commun. Math. Phys.* **73** 105
[5] Campbell M 2001 *Commun. Math. Phys.* **218** 99
[6] Hastings M B 1999 *Phys. Rev.* B **60** 9755
[7] John S 1990 *Scattering and Localization of Classical Waves in Random Media* ed P Sheng (Singapore: World Scientific) p 1